Math 53 – Worksheet 2

GSI: Oltman, (2/5/19)

12.3 - 12.5 (dot product, cross product, lines and planes)

Problem 1. *If $a = \langle 3, 0, -1 \rangle$, find a vector b such that the magnitude of the projection of b onto a is 2.*

Solution 1. We would like:

$$2 = \frac{b \cdot a}{\|a\|}$$

if $b = (x, y, z)$ then we have:

$$\frac{3x - z}{\sqrt{10}} = 2$$

this has infinite solutions, if we want just one, we can set $x = y = 0$ and let $z = -2\sqrt{10}$

Problem 2. *Compute $(i \times j) \times k$*

Solution 2. $i \times j = k$ therefore $(i \times j) \times k = k \times k = 0$

Problem 3. *If $a = \langle 2, -1, 3 \rangle$ and $b = \langle 4, 2, 1 \rangle$, compute $a \times b$ and $b \times a$*

Solution 3. $(-7, 10, 8)$

Problem 4. *Find the area of the parallelogram with vertices $A(-3, 0), B(-1, 3), C(5, 2), D(3, -1)$*

Solution 4. D is not necessary. We need two vectors found by going from A to B and A to C, this gives the vectors $(-4, 3)$ and $(8, 2)$. To find the area of the parallelogram, we need to have these be in 3 dimensions, so we just let $z = 0$ for both these and take the magnitude of the cross product;

$$|(-4, 3, 0) \times (8, 2, 0)| = 16$$

Problem 5. *Is the line through $(-4, -6, 1)$ and $(-2, 0, -3)$ parallel to the the line from $(10, 18, 4)$ and $(5, 3, 14)$?*

Solution 5. Take the difference to get the vectors $(2, 6, -4)$ and $(-5, -15, 10)$, then you can either see these are multiples of each other, or see that their cross product is the zero vector which says that they are parallel.

Problem 6. *Find the equation of the plane through the origin and the points $(3, -2, 1)$ and $(1, 1, 1)$*

Solution 6. We have the origin is included in the plane, we just need the normal, which is found by dotting the two vectors:

$$(3, -2, 1) \times (1, 1, 1) = (-3, -2, 5)$$

therefore the equation for the plane is $-3x - 2y + 5z = 0$

Problem 7. *Find the distance between the given parallel planes:*

$$2x - 3y + z = 4$$
$$4x - 6y + 2z = 3$$

Solution 7. There is a formula in the book that gives the distance between planes. My solution is to find the equation for a line normal to the planes that intersects the two planes, find the two intersection points and take the magnitude. The normal is $(2, -3, 1)$ and the first plane contains the point $(0, 0, 4)$. We therefore have the line $r(t) = (2t, -3t, 4 + t)$. We can find the intersection of the second plane by plugging in this set of points into the second equation:

$$4(2t) - 6(-3t) + 2(4 + t) = 0$$

we solve for t which I got is 5/6 (could be wrong). We therefore have the point on the other plane as $(5/3, -5/2, 4 + 5/6)$. Therefore the distance would be $\|(5/3, -5/2, 5/6\|$